Kuwait University — Math 101 — Date: Dec. 15, 2005
Dept. of Math. & Comp. Sci. — Second Exam — Duration: 90 minutes

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed

Answer the following questions:

1. Use differentials to approximate $\sqrt[4]{15.9}$. (3 pts.)

2. Find an equation of the normal line at $x = 0$, to the graph of
$$\sec^2(\pi + x) + \sin(xy) + y = 0.$$ (4 pts.)

3. (a) State The Mean Value Theorem. (1 pt.)

   (b) Use The Mean Value Theorem to show that
$$(1+x)^{\frac{2}{3}} < 4 + \tfrac{2}{3}(x-7), \quad \text{for every } x > 7.$$
   [Hint: take $f(x) = (1+x)^{\frac{2}{3}}$] (3 pts.)

4. A plate in a shape of a square is heated. If the area A of the plate (in cm^2) after time t (in hours) is given by
$$A = \sqrt{1+t^3},$$
   find the rate at which the sides of the plate are changing after *two* hours. (4 pts.)

5. Let $f(x) = \dfrac{x(3x-8)}{(x-2)^2}$, and given that $f'(x) = \dfrac{4(4-x)}{(x-2)^3}$ and $f''(x) = \dfrac{8(x-5)}{(x-2)^4}$.

   (a) Find the vertical and horizontal asymptotes for the graph of f, if any.

   (b) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find the local extrema of f, if any.

   (c) Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the points of inflection, if any.

   (d) Sketch the graph of f.

   (e) Find the maximum and the minimum values of f on $[3, 5]$. (10 pts.)